9.12 A 50 wt% Pb-50 wt% Mg alloy is slowly cooled from 700°C (1290°F) to 400°C (750°F).

(a) At what temperature does the first solid phase form?
(b) What is the composition of this solid phase?
(c) At what temperature does the liquid solidify?
(d) What is the composition of this last remaining liquid phase?

Solution

Shown below is the Mg-Pb phase diagram (Figure 9.20) and a vertical line constructed at a composition of 50 wt% Pb-50 wt% Mg.

(a) Upon cooling from 700°C, the first solid phase forms at the temperature at which a vertical line at this composition intersects the $L-(\alpha + L)$ phase boundary--i.e., about 560°C;

(b) The composition of this solid phase corresponds to the intersection with the $\alpha-(\alpha + L)$ phase boundary, of a tie line constructed across the $\alpha + L$ phase region at 560°C--i.e., 21 wt% Pb-79 wt% Mg;

(c) Complete solidification of the alloy occurs at the intersection of this same vertical line at 50 wt% Pb with the eutectic isotherm--i.e., about 465°C;

(d) The composition of the last liquid phase remaining prior to complete solidification corresponds to the eutectic composition--i.e., about 67 wt% Pb-33 wt% Mg.
9.13 For an alloy of composition 74 wt% Zn-26 wt% Cu, cite the phases present and their compositions at the following temperatures: 850°C, 750°C, 680°C, 600°C, and 500°C.

Solution

This problem asks us to determine the phases present and their concentrations at several temperatures, for an alloy of composition 74 wt% Zn-26 wt% Cu. From Figure 9.19 (the Cu-Zn phase diagram), which is shown below with a vertical line constructed at the specified composition:

At 850°C, a liquid phase is present; \( C_L = 74 \text{ wt}\% \text{ Zn-26 wt}\% \text{ Cu} \)

At 750°C, \( \gamma \) and liquid phases are present; \( C_{\gamma} = 67 \text{ wt}\% \text{ Zn-33 wt}\% \text{ Cu}; C_L = 77 \text{ wt}\% \text{ Zn-23 wt}\% \text{ Cu} \)

At 680°C, \( \delta \) and liquid phases are present; \( C_{\delta} = 73 \text{ wt}\% \text{ Zn-27 wt}\% \text{ Cu}; C_L = 82 \text{ wt}\% \text{ Zn-18 wt}\% \text{ Cu} \)

At 600°C, the \( \delta \) phase is present; \( C_{\delta} = 74 \text{ wt}\% \text{ Zn-26 wt}\% \text{ Cu} \)

At 500°C, \( \gamma \) and \( \epsilon \) phases are present; \( C_{\gamma} = 69 \text{ wt}\% \text{ Zn-31 wt}\% \text{ Cu}; C_{\epsilon} = 78 \text{ wt}\% \text{ Zn-22 wt}\% \text{ Cu} \)
A 90 wt% Ag-10 wt% Cu alloy is heated to a temperature within the $\beta +$ liquid phase region. If the composition of the liquid phase is 85 wt% Ag, determine:

(a) The temperature of the alloy
(b) The composition of the $\beta$ phase
(c) The mass fractions of both phases

**Solution**

(a) In order to determine the temperature of a 90 wt% Ag-10 wt% Cu alloy for which $\beta$ and liquid phases are present with the liquid phase of composition 85 wt% Ag, we need to construct a tie line across the $\beta + L$ phase region of Figure 9.7 that intersects the liquidus line at 85 wt% Ag; this is possible at about 850°C.

(b) The composition of the $\beta$ phase at this temperature is determined from the intersection of this same tie line with solidus line, which corresponds to about 95 wt% Ag.

(c) The mass fractions of the two phases are determined using the lever rule, Equations 9.1 and 9.2 with $C_0 = 90$ wt% Ag, $C_L = 85$ wt% Ag, and $C_\beta = 95$ wt% Ag, as

\[
W_\beta = \frac{C_0 - C_L}{C_\beta - C_L} = \frac{90 - 85}{95 - 85} = 0.50
\]

\[
W_L = \frac{C_\beta - C_0}{C_\beta - C_L} = \frac{95 - 90}{95 - 85} = 0.50
\]
9.38 On the basis of the photomicrograph (i.e., the relative amounts of the microconstituents) for the lead–tin alloy shown in Figure 9.17 and the Pb–Sn phase diagram (Figure 9.8), estimate the composition of the alloy, and then compare this estimate with the composition given in the figure legend of Figure 9.17. Make the following assumptions: (1) the area fraction of each phase and microconstituent in the photomicrograph is equal to its volume fraction; (2) the densities of the α and β phases as well as the eutectic structure are 11.2, 7.3, and 8.7 g/cm³, respectively; and (3) this photomicrograph represents the equilibrium microstructure at 180°C (355°F).

Solution

Below is shown the micrograph of the Pb-Sn alloy, Figure 9.17:

Primary α and eutectic microconstituents are present in the photomicrograph, and it is given that their densities are 11.2 and 8.7 g/cm³, respectively. Below is shown a square grid network onto which is superimposed outlines of the primary α phase areas.
The area fraction of this primary $\alpha$ phase may be determined by counting squares. There are a total of 644 squares, and of these, approximately 104 lie within the primary $\alpha$ phase particles. Thus, the area fraction of primary $\alpha$ is $104/644 = 0.16$, which is also assumed to be the volume fraction.

We now want to convert the volume fractions into mass fractions in order to employ the lever rule to the Pb-Sn phase diagram. To do this, it is necessary to utilize Equations 9.7a and 9.7b as follows:

$$W_{\alpha'} = \frac{V_{\alpha'} \rho_{\alpha'}}{V_{\alpha'} \rho_{\alpha'} + V_{\text{eutectic}} \rho_{\text{eutectic}}}$$

$$= \frac{(0.16)(11.2 \text{ g/cm}^3)}{(0.16)(11.2 \text{ g/cm}^3) + (0.84)(8.7 \text{ g/cm}^3)} = 0.197$$

$$W_{\text{eutectic}} = \frac{V_{\text{eutectic}} \rho_{\text{eutectic}}}{V_{\alpha'} \rho_{\alpha'} + V_{\text{eutectic}} \rho_{\text{eutectic}}}$$

$$= \frac{(0.84)(8.7 \text{ g/cm}^3)}{(0.16)(11.2 \text{ g/cm}^3) + (0.84)(8.7 \text{ g/cm}^3)} = 0.803$$

From Figure 9.8, we want to use the lever rule and a tie-line that extends from the eutectic composition (61.9 wt% Sn) to the $\alpha$–($\alpha + \beta$) phase boundary at 180°C (about 18.3 wt% Sn). Accordingly

$$W_{\alpha'} = 0.197 = \frac{61.9 - C_0}{61.9 - 18.3}$$

wherein $C_0$ is the alloy composition (in wt% Sn). Solving for $C_0$ yields $C_0 = 53.3$ wt% Sn. This value is in good agreement with the actual composition—viz. 50 wt% Sn.
Two intermetallic compounds, $AB$ and $AB_2$, exist for elements A and B. If the compositions for $AB$ and $AB_2$ are 34.3 wt% A–65.7 wt% B and 20.7 wt% A–79.3 wt% B, respectively, and element A is potassium, identify element B.

Solution

This problem gives us the compositions in weight percent for the two intermetallic compounds $AB$ and $AB_2$, and then asks us to identify element B if element A is potassium. Probably the easiest way to solve this problem is to first compute the ratio of the atomic weights of these two elements using Equation 4.6a; then, since we know the atomic weight of potassium (39.10 g/mol, per inside the front cover), it is possible to determine the atomic weight of element B, from which an identification may be made.

First of all, consider the $AB$ intermetallic compound; inasmuch as it contains the same numbers of A and B atoms, its composition in atomic percent is 50 at% A–50 at% B. Equation 4.6a may be written in the form:

$$C_B' = \frac{C_B A_A}{C_A A_B + C_B A_A} \times 100$$

where $A_A$ and $A_B$ are the atomic weights for elements A and B, and $C_A$ and $C_B$ are their compositions in weight percent. For this $AB$ compound, and making the appropriate substitutions in the above equation leads to

$$50 \text{ at } % \ B = \frac{(65.7 \ \text{wt} \ % \ B)(A_A)}{(34.3 \ \text{wt} \ % \ A)(A_B) + (65.7 \ \text{wt} \ % \ B)(A_A)} \times 100$$

Now, solving this expression yields,

$$A_B = 1.916 A_A$$

Since potassium is element A and it has an atomic weight of 39.10 g/mol, the atomic weight of element B is just

$$A_B = (1.916)(39.10 \text{ g/mol}) = 74.92 \text{ g/mol}$$

Upon consultation of the period table of the elements (Figure 2.6) we note the element that has an atomic weight closest to this value is arsenic (74.92 g/mol). Therefore, element B is arsenic, and the two intermetallic compounds are KAs and KAs$_2$. 
9.43 Figure 9.37 is a portion of the titanium-copper phase diagram for which only single-phase regions are labeled. Specify all temperature-composition points at which eutectics, eutectoids, peritectics, and congruent phase transformations occur. Also, for each, write the reaction upon cooling.

Solution

Below is shown the titanium-copper phase diagram (Figure 9.37).

There is one eutectic on this phase diagram, which exists at about 51 wt% Cu-49 wt% Ti and 960°C. Its reaction upon cooling is

\[ L \rightarrow \text{Ti}_2\text{Cu} + \text{TiCu} \]

There is one eutectoid for this system. It exists at about 7.5 wt% Cu-92.5 wt% Ti and 790°C. This reaction upon cooling is

\[ \beta \rightarrow \alpha + \text{Ti}_2\text{Cu} \]

There is one peritectic on this phase diagram. It exists at about 40 wt% Cu-60 wt% Ti and 1005°C. The reaction upon cooling is
β + L → Ti₂Cu

There is a single congruent melting point that exists at about 57.5 wt% Cu-42.5 wt% Ti and 982°C. The reaction upon cooling is

L → TiCu
9.47 (a) What is the distinction between hypoeutectoid and hypereutectoid steels?  
(b) In a hypoeutectoid steel, both eutectoid and proeutectoid ferrite exist. Explain the difference between them. What will be the carbon concentration in each?  

Solution  

(a) A “hypoeutectoid” steel has a carbon concentration less than the eutectoid; on the other hand, a “hypereutectoid” steel has a carbon content greater than the eutectoid.  
(b) For a hypoeutectoid steel, the proeutectoid ferrite is a microconstituent that formed above the eutectoid temperature. The eutectoid ferrite is one of the constituents of pearlite that formed at a temperature below the eutectoid. The carbon concentration for both ferrites is 0.022 wt% C.
9.48 What is the carbon concentration of an iron-carbon alloy for which the fraction of total ferrite is 0.94?

Solution

This problem asks that we compute the carbon concentration of an iron-carbon alloy for which the fraction of total ferrite is 0.94. Application of the lever rule (of the form of Equation 9.12) yields

\[ W_\alpha = 0.94 = \frac{C_{\text{Fe}_3\text{C}} - C_0^*}{C_{\text{Fe}_3\text{C}} - C_\alpha} = \frac{6.70 - C_0^*}{6.70 - 0.022} \]

and solving for \( C_0^* \)

\[ C_0^* = 0.42 \text{ wt\% C} \]
9.51 Consider 2.5 kg of austenite containing 0.65 wt% C, cooled to below 727°C (1341°F).

(a) What is the proeutectoid phase?

(b) How many kilograms each of total ferrite and cementite form?

(c) How many kilograms each of pearlite and the proeutectoid phase form?

(d) Schematically sketch and label the resulting microstructure.

Solution

(a) Ferrite is the proeutectoid phase since 0.65 wt% C is less than 0.76 wt% C.

(b) For this portion of the problem, we are asked to determine how much total ferrite and cementite form. For ferrite, application of the appropriate lever rule expression yields

\[ W_\alpha = \frac{C_{Fe_3C} - C_0}{C_{Fe_3C} - C_\alpha} = \frac{6.70 - 0.65}{6.70 - 0.022} = 0.91 \]

which corresponds to \((0.91)(2.5 \text{ kg}) = 2.27 \text{ kg}\) of total ferrite.

Similarly, for total cementite,

\[ W_{Fe_3C} = \frac{C_0 - C_\alpha}{C_{Fe_3C} - C_\alpha} = \frac{0.65 - 0.022}{6.70 - 0.022} = 0.09 \]

Or \((0.09)(2.5 \text{ kg}) = 0.23 \text{ kg}\) of total cementite form.

(c) Now consider the amounts of pearlite and proeutectoid ferrite. Using Equation 9.20

\[ W_p = \frac{C'_0 - 0.022}{0.74} = \frac{0.65 - 0.022}{0.74} = 0.85 \]

This corresponds to \((0.85)(2.5 \text{ kg}) = 2.12 \text{ kg}\) of pearlite.

Also, from Equation 9.21,

\[ W_\alpha' = \frac{0.76 - 0.65}{0.74} = 0.15 \]

Or, there are \((0.15)(2.5 \text{ kg}) = 0.38 \text{ kg}\) of proeutectoid ferrite.

(d) Schematically, the microstructure would appear as: