12.3 *Show that the minimum cation-to-anion radius ratio for a coordination number of 6 is 0.414. [Hint: Use the NaCl crystal structure (Figure 12.2), and assume that anions and cations are just touching along cube edges and across face diagonals.]*

**Solution**

This problem asks us to show that the minimum cation-to-anion radius ratio for a coordination number of 6 is 0.414 (using the rock salt crystal structure). Below is shown one of the faces of the rock salt crystal structure in which anions and cations just touch along the edges, and also the face diagonals.

From triangle $FGH$,

$$GF = 2r_A$$

and

$$FH = GH = r_A + r_C$$

Since $FGH$ is a right triangle

$$(GH)^2 + (FH)^2 = (FG)^2$$

or

$$(r_A + r_C)^2 + (r_A + r_C)^2 = (2r_A)^2$$

which leads to
\[ r_A + r_C = \frac{2r_A}{\sqrt{2}} \]

Or, solving for \( r_C/r_A \)

\[ \frac{r_C}{r_A} = \left( \frac{2}{\sqrt{2}} - 1 \right) = 0.414 \]
12.4 Demonstrate that the minimum cation-to-anion radius ratio for a coordination number of 8 is 0.732.

Solution

This problem asks us to show that the minimum cation-to-anion radius ratio for a coordination number of 8 is 0.732. From the cubic unit cell shown below

the unit cell edge length is $2r_A$, and from the base of the unit cell

\[ x^2 = (2r_A)^2 + (2r_A)^2 = 8r_A^2 \]

Or

\[ x = 2r_A\sqrt{2} \]

Now from the triangle that involves $x$, $y$, and the unit cell edge

\[ x^2 + (2r_A)^2 = y^2 = (2r_A + 2r_C)^2 \]

\[ (2r_A\sqrt{2})^2 + 4r_A^2 = (2r_A + 2r_C)^2 \]

Which reduces to

\[ 2r_A(\sqrt{3} - 1) = 2r_C \]

Or

\[ \frac{r_C}{r_A} = \sqrt{3} - 1 = 0.732 \]
On the basis of ionic charge and ionic radii given in Table 12.3, predict crystal structures for the following materials:

(a) CsI,
(b) NiO,
(c) KI, and
(d) NiS.

Justify your selections.

Solution

This problem calls for us to predict crystal structures for several ceramic materials on the basis of ionic charge and ionic radii.

(a) For CsI, using data from Table 12.3

\[
\frac{r_{\text{Cs}^+}}{r_{\text{I}^-}} = \frac{0.170 \text{ nm}}{0.220 \text{ nm}} = 0.773
\]

Now, from Table 12.2, the coordination number for each cation (Cs\(^{+}\)) is eight, and, using Table 12.4, the predicted crystal structure is cesium chloride.

(b) For NiO, using data from Table 12.3

\[
\frac{r_{\text{Ni}^{2+}}}{r_{\text{O}^{2-}}} = \frac{0.069 \text{ nm}}{0.140 \text{ nm}} = 0.493
\]

The coordination number is six (Table 12.2), and the predicted crystal structure is sodium chloride (Table 12.4).

(c) For KI, using data from Table 12.3

\[
\frac{r_{\text{K}^+}}{r_{\text{I}^-}} = \frac{0.138 \text{ nm}}{0.220 \text{ nm}} = 0.627
\]

The coordination number is six (Table 12.2), and the predicted crystal structure is sodium chloride (Table 12.4).

(d) For NiS, using data from Table 12.3

\[
\frac{r_{\text{Ni}^{2+}}}{r_{\text{S}^{2-}}} = \frac{0.069 \text{ nm}}{0.184 \text{ nm}} = 0.375
\]

The coordination number is four (Table 12.2), and the predicted crystal structure is zinc blende (Table 12.4).
12.8 **The zinc blende crystal structure is one that may be generated from close-packed planes of anions.**

(a) *Will the stacking sequence for this structure be FCC or HCP? Why?*

(b) *Will cations fill tetrahedral or octahedral positions? Why?*

(c) *What fraction of the positions will be occupied?*

**Solution**

This question is concerned with the zinc blende crystal structure in terms of close-packed planes of anions.

(a) The stacking sequence of close-packed planes of anions for the zinc blende crystal structure will be the same as FCC (and not HCP) because the anion packing is FCC (Table 12.4).

(b) The cations will fill tetrahedral positions since the coordination number for cations is four (Table 12.4).

(c) Only one-half of the tetrahedral positions will be occupied because there are two tetrahedral sites per anion, and yet only one cation per anion.
12.10 Iron sulfide (FeS) may form a crystal structure that consists of an HCP arrangement of \( S^2^- \) ions.

(a) Which type of interstitial site will the \( Fe^{2+} \) ions occupy?

(b) What fraction of these available interstitial sites will be occupied by \( Fe^{2+} \) ions?

**Solution**

(a) This portion of the problem asks that we specify which type of interstitial site the \( Fe^{2+} \) ions will occupy in FeS if the \( S^2^- \) ions form an HCP arrangement. Since, from Table 12.3, \( r_{S^2-} = 0.184 \) nm and \( r_{Fe^{2+}} = 0.077 \) nm, then

\[
\frac{r_{Fe^{2+}}}{r_{S^2-}} = \frac{0.077 \text{ nm}}{0.184 \text{ nm}} = 0.418
\]

Inasmuch as \( r_C/r_A \) is between 0.414 and 0.732, the coordination number for \( Fe^{2+} \) is 6 (Table 12.2); therefore, tetrahedral octahedral positions are occupied.

(b) We are now asked what fraction of these available interstitial sites are occupied by \( Fe^{2+} \) ions. Since there is 1 octahedral site per \( S^2^- \) ion, and the ratio of \( Fe^{2+} \) to \( S^2^- \) is 1:1, all of these sites are occupied with \( Fe^{2+} \) ions.
12.13 Calculate the density of FeO, given that it has the rock salt crystal structure.

Solution

We are asked to calculate the theoretical density of FeO. This density may be computed using Equation (12.1) as

$$\rho = \frac{n'(A_{Fe} + A_{O})}{V_C N_A}$$

Since the crystal structure is rock salt, $n' = 4$ formula units per unit cell. Using the ionic radii for Fe$^{2+}$ and O$^{2-}$ from Table 12.3, the unit cell volume is computed as follows:

$$V_C = a^3 = (2r_{Fe^{2+}} + 2r_{O^{2-}})^3 = [2(0.077 \text{ nm}) + 2(0.140 \text{ nm})]^3$$

$$= 0.0817 \text{ nm}^3/\text{unit cell} = 8.17 \times 10^{-23} \text{ cm}^3/\text{unit cell}$$

Thus,

$$\rho = \frac{(4 \text{ formula units/unit cell})(55.85 \text{ g/mol} + 16.00 \text{ g/mol})}{(8.17 \times 10^{-23} \text{ cm}^3/\text{unit cell})(6.022 \times 10^{23} \text{ formula units/mol})}$$

$$= 5.84 \text{ g/cm}^3$$
A hypothetical AX type of ceramic material is known to have a density of 2.65 g/cm$^3$ and a unit cell of cubic symmetry with a cell edge length of 0.43 nm. The atomic weights of the A and X elements are 86.6 and 40.3 g/mol, respectively. On the basis of this information, which of the following crystal structures is (are) possible for this material: rock salt, cesium chloride, or zinc blende? Justify your choice(s).

**Solution**

We are asked to specify possible crystal structures for an AX type of ceramic material given its density (2.65 g/cm$^3$), that the unit cell has cubic symmetry with edge length of 0.43 nm (4.3 × 10$^{-8}$ cm), and the atomic weights of the A and X elements (86.6 and 40.3 g/mol, respectively). Using Equation 12.1 and solving for $n'$ yields

$$n' = \frac{\rho V \cdot N_A}{\sum A_C + \sum A_A}$$

$$= \frac{(2.65 \text{ g/cm}^3) \left[(4.30 \times 10^{-8} \text{ cm})^3/\text{unit cell}\right] (6.022 \times 10^{23} \text{ formula units/mol})}{(86.6 + 40.3) \text{ g/mol}}$$

$$= 1.00 \text{ formula units/unit cell}$$

Of the three possible crystal structures, only cesium chloride has one formula unit per unit cell, and therefore, is the only possibility.
9.1 Consider the sugar–water phase diagram of Figure 9.1.

(a) How much sugar will dissolve in 1500 g water at 90 °C (194 °F)?

(b) If the saturated liquid solution in part (a) is cooled to 20 °C (68 °F), some of the sugar will precipitate out as a solid. What will be the composition of the saturated liquid solution (in wt% sugar) at 20 °C?

(c) How much of the solid sugar will come out of solution upon cooling to 20 °C?

Solution

(a) We are asked to determine how much sugar will dissolve in 1000 g of water at 90 °C. From the solubility limit curve in Figure 9.1, at 90 °C the maximum concentration of sugar in the syrup is about 77 wt%. It is now possible to calculate the mass of sugar using Equation 4.3 as

\[
C_{\text{sugar (wt\%)}} = \frac{m_{\text{sugar}}}{m_{\text{sugar}} + m_{\text{water}}} \times 100
\]

\[
77 \text{ wt\%} = \frac{m_{\text{sugar}}}{m_{\text{sugar}} + 1500 \text{ g}} \times 100
\]

Solving for \(m_{\text{sugar}}\) yields \(m_{\text{sugar}} = 5022 \text{ g}\)

(b) Again using this same plot, at 20 °C the solubility limit (or the concentration of the saturated solution) is about 64 wt% sugar.

(c) The mass of sugar in this saturated solution at 20 °C (\(m'_{\text{sugar}}\)) may also be calculated using Equation 4.3 as follows:

\[
64 \text{ wt\%} = \frac{m'_{\text{sugar}}}{m'_{\text{sugar}} + 1500 \text{ g}} \times 100
\]

which yields a value for \(m'_{\text{sugar}}\) of 2667 g. Subtracting the latter from the former of these sugar concentrations yields the amount of sugar that precipitated out of the solution upon cooling \(m''_{\text{sugar}}\), that is
9.3  *Cite three variables that determine the microstructure of an alloy.*

**Solution**

Three variables that determine the microstructure of an alloy are (1) the alloying elements present, (2) the concentrations of these alloying elements, and (3) the heat treatment of the alloy.

9.6  *At a pressure of 0.01 atm, determine (a) the melting temperature for ice, and (b) the boiling temperature for water.*

**Solution**

The melting temperature for ice and the boiling temperature for water at a pressure of 0.01 atm may be determined from the pressure-temperature diagram for this system, Figure 10.2, which is shown below; a horizontal line has been constructed across this diagram at a pressure of 0.01 atm.

![Pressure-Temperature Diagram](image)

The melting and boiling temperatures for ice at a pressure of 0.01 atm may be determined by moving horizontally across the pressure-temperature diagram at this pressure. The temperature corresponding to the intersection of the Ice-Liquid phase boundary is the melting temperature, which is approximately 1°C. On the other hand, the boiling temperature is at the intersection of the horizontal line with the Liquid-Vapor phase boundary—approximately 16°C.
9.9 *Is it possible to have a copper–nickel alloy that, at equilibrium, consists of a liquid phase of composition 20 wt% Ni–80 wt% Cu and also an α phase of composition 37 wt% Ni–63 wt% Cu? If so, what will be the approximate temperature of the alloy? If this is not possible, explain why.*

**Solution**

It is *not possible* to have a Cu-Ni alloy, which at equilibrium, consists of a liquid phase of composition 20 wt% Ni-80 wt% Cu and an α phase of composition 37 wt% Ni-63 wt% Cu. From Figure 9.3a, a single tie line does not exist within the α + L region that intersects the phase boundaries at the given compositions. At 20 wt% Ni, the $L-(\alpha + L)$ phase boundary is at about 1200°C, whereas at 37 wt% Ni the $(L + \alpha)$-α phase boundary is at about 1230°C.
9.11 A copper-nickel alloy of composition 70 wt% Ni-30 wt% Cu is slowly heated from a temperature of 1300°C (2370°F).

(a) At what temperature does the first liquid phase form?
(b) What is the composition of this liquid phase?
(c) At what temperature does complete melting of the alloy occur?
(d) What is the composition of the last solid remaining prior to complete melting?

Solution

Shown below is the Cu-Ni phase diagram (Figure 9.3a) and a vertical line constructed at a composition of 70 wt% Ni-30 wt% Cu.

(a) Upon heating from 1300°C, the first liquid phase forms at the temperature at which this vertical line intersects the $\alpha$-($\alpha + L$) phase boundary--i.e., about 1345°C.

(b) The composition of this liquid phase corresponds to the intersection with the ($\alpha + L$)-$L$ phase boundary, of a tie line constructed across the $\alpha + L$ phase region at 1345°C--i.e., 59 wt% Ni;

(c) Complete melting of the alloy occurs at the intersection of this same vertical line at 70 wt% Ni with the ($\alpha + L$)-$L$ phase boundary--i.e., about 1380°C;
(d) The composition of the last solid remaining prior to complete melting corresponds to the intersection with $\alpha-(\alpha + L)$ phase boundary, of the tie line constructed across the $\alpha + L$ phase region at 1380°C--i.e., about 79 wt% Ni.
9.15 A 1.5-kg specimen of a 90 wt% Pb–10 wt% Sn alloy is heated to 250°C (480°F); at this temperature it is entirely an α-phase solid solution (Figure 9.8). The alloy is to be melted to the extent that 50% of the specimen is liquid, the remainder being the α phase. This may be accomplished either by heating the alloy or changing its composition while holding the temperature constant.

(a) To what temperature must the specimen be heated?

(b) How much tin must be added to the 1.5-kg specimen at 250°C to achieve this state?

Solution

(a) Probably the easiest way to solve this part of the problem is by trial and error—that is, on the Pb-Sn phase diagram (Figure 9.8), moving vertically at the given composition, through the α + L region until the tie-line lengths on both sides of the given composition are the same. This occurs at approximately 295°C (560°F).

(b) We can also produce a 50% liquid solution at 250°C, by adding Sn to the alloy. At 250°C and within the α + L phase region

\[ C_\alpha = 14 \text{ wt\% Sn-86 wt\% Pb} \]
\[ C_L = 34 \text{ wt\% Sn-66 wt\% Pb} \]

Let \( C_0 \) be the new alloy composition to give \( W_\alpha = W_L = 0.5 \). Then,

\[ W_\alpha = 0.5 = \frac{C_L - C_0}{C_L - C_\alpha} = \frac{34 - C_0}{34 - 14} \]

And solving for \( C_0 \) gives 24 wt% Sn. Now, let \( m_{Sn} \) be the mass of Sn added to the alloy to achieve this new composition. The amount of Sn in the original alloy is

\[ (0.10)(1.5 \text{ kg}) = 0.15 \text{ kg} \]

Then, using a modified form of Equation 4.3

\[ \left( \frac{0.15 \text{ kg} + m_{Sn}}{1.5 \text{ kg} + m_{Sn}} \right) \times 100 = 24 \]

And, solving for \( m_{Sn} \) (the mass of tin to be added), yields \( m_{Sn} = 0.276 \text{ kg} \).
9.20 A hypothetical A–B alloy of composition 55 wt% B–45 wt% A at some temperature is found to consist of mass fractions of 0.5 for both α and β phases. If the composition of the β phase is 90 wt% B–10 wt% A, what is the composition of the α phase?

Solution

For this problem, we are asked to determine the composition of the β phase given that

\[ C_0 = 55 \text{ (or } 55 \text{ wt% B–45 wt% A)} \]

\[ C_\beta = 90 \text{ (or } 90 \text{ wt% B–10 wt% A)} \]

\[ W_\alpha = W_\beta = 0.5 \]

If we set up the lever rule for \( W_\alpha \)

\[ W_\alpha = 0.5 = \frac{C_\beta - C_0}{C_\beta - C_\alpha} = \frac{90 - 55}{90 - C_\alpha} \]

And solving for \( C_\alpha \)

\[ C_\alpha = 20 \text{ (or } 20 \text{ wt% B–80 wt% A)} \]
9.24 Determine the relative amounts (in terms of volume fractions) of the phases for the alloys and temperatures given in Problem 9.8a, b, and c. Below are given the approximate densities of the various metals at the alloy temperatures:

<table>
<thead>
<tr>
<th>Metal</th>
<th>Temperature (°C)</th>
<th>Density (g/cm³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ag</td>
<td>900</td>
<td>9.97</td>
</tr>
<tr>
<td>Cu</td>
<td>400</td>
<td>8.77</td>
</tr>
<tr>
<td>Cu</td>
<td>900</td>
<td>8.56</td>
</tr>
<tr>
<td>Pb</td>
<td>175</td>
<td>11.20</td>
</tr>
<tr>
<td>Sn</td>
<td>175</td>
<td>7.22</td>
</tr>
<tr>
<td>Zn</td>
<td>400</td>
<td>6.83</td>
</tr>
</tbody>
</table>

Solution

This problem asks that we determine the phase volume fractions for the alloys and temperatures in Problems 9.8a, b, and c. This is accomplished by using the technique illustrated in Example Problem 9.3, and also the results of Problems 9.8 and 9.14.

(a) This is a Cu-Zn alloy at 400°C, wherein

\[ C_\varepsilon = 87 \text{ wt}\% \text{ Zn} - 13 \text{ wt}\% \text{ Cu} \]
\[ C_\eta = 97 \text{ wt}\% \text{ Zn} - 3 \text{ wt}\% \text{ Cu} \]
\[ W_\varepsilon = 0.70 \]
\[ W_\eta = 0.30 \]
\[ \rho_{Cu} = 8.77 \text{ g/cm}^3 \]
\[ \rho_{Zn} = 6.83 \text{ g/cm}^3 \]

Using these data it is first necessary to compute the densities of the \( \varepsilon \) and \( \eta \) phases using Equation 4.10a. Thus

\[ \rho_\varepsilon = \frac{100}{\frac{C_{Zn}(\varepsilon)}{\rho_{Zn}} + \frac{C_{Cu}(\varepsilon)}{\rho_{Cu}}} \]

\[ = \frac{100}{\frac{87}{6.83 \text{ g/cm}^3} + \frac{13}{8.77 \text{ g/cm}^3}} = 7.03 \text{ g/cm}^3 \]
\[ \rho_\eta = \frac{100}{\frac{C_{Zn(\eta)}}{\rho_{Zn}} + \frac{C_{Cu(\eta)}}{\rho_{Cu}}} \]

\[ = \frac{100}{6.83 \text{ g/cm}^3 + \frac{3}{8.77 \text{ g/cm}^3}} = 6.88 \text{ g/cm}^3 \]

Now we may determine the \( V_\epsilon \) and \( V_\eta \) values using Equation 9.6. Thus,

\[ V_\epsilon = \frac{W_\epsilon}{\rho_\epsilon} = \frac{W_\epsilon}{\rho_\epsilon + \rho_\eta} \]

\[ = \frac{0.70}{7.03 \text{ g/cm}^3} = 0.70 \]

\[ V_\eta = \frac{W_\eta}{\rho_\eta} = \frac{W_\eta}{\rho_\epsilon + \rho_\eta} \]

\[ = \frac{0.30}{6.88 \text{ g/cm}^3} = 0.30 \]

(b) This is a Pb-Sn alloy at 175°C, wherein

\( C_\alpha = 16 \text{ wt\% Sn-84 wt\% Pb} \)

\( C_\beta = 97 \text{ wt\% Sn-3 wt\% Pb} \)

\( W_\alpha = 0.27 \)

\( W_\beta = 0.73 \)

\( \rho_{\text{Sn}} = 7.22 \text{ g/cm}^3 \)
\( \rho_{\text{Pb}} = 11.20 \, \text{g/cm}^3 \)

Using this data it is first necessary to compute the densities of the \( \alpha \) and \( \beta \) phases. Thus

\[
\rho_\alpha = \frac{100}{\frac{C_{\text{Sn} (\alpha)}}{\rho_{\text{Sn}}} + \frac{C_{\text{Pb} (\alpha)}}{\rho_{\text{Pb}}}}
\]

\[
= \frac{100}{\frac{16}{7.22 \, \text{g/cm}^3} + \frac{84}{11.20 \, \text{g/cm}^3}} = 10.29 \, \text{g/cm}^3
\]

\[
\rho_\beta = \frac{100}{\frac{C_{\text{Sn} (\beta)}}{\rho_{\text{Sn}}} + \frac{C_{\text{Pb} (\beta)}}{\rho_{\text{Pb}}}}
\]

\[
= \frac{100}{\frac{97}{7.22 \, \text{g/cm}^3} + \frac{3}{11.20 \, \text{g/cm}^3}} = 7.30 \, \text{g/cm}^3
\]

Now we may determine the \( V_\alpha \) and \( V_\beta \) values using Equation 9.6. Thus,

\[
V_\alpha = \frac{W_\alpha}{\rho_\alpha} \frac{W_\alpha}{W_\alpha + W_\beta} \rho_\alpha \rho_\beta
\]

\[
= \frac{0.27}{10.29 \, \text{g/cm}^3} \frac{0.27}{0.27 + 0.73} = 0.21
\]

\[
V_\beta = \frac{W_\beta}{\rho_\beta} \frac{W_\beta}{W_\alpha + W_\beta} \rho_\alpha \rho_\beta
\]
\[\frac{0.73}{7.30 \text{ g/cm}^3} = \frac{0.27}{10.29 \text{ g/cm}^3} + \frac{0.73}{7.30 \text{ g/cm}^3} = 0.79\]

(c) This is a Ag-Cu alloy at 900°C, wherein only the liquid phase is present. Therefore, \(V_L = 1.0\).
9.33 The microstructure of a lead-tin alloy at 180°C (355°F) consists of primary β and eutectic structures. If the mass fractions of these two microconstituents are 0.57 and 0.43, respectively, determine the composition of the alloy.

Solution

Since there is a primary β microconstituent present, then we know that the alloy composition, $C_0$ is between 61.9 and 97.8 wt% Sn (Figure 9.8). Furthermore, this figure also indicates that $C_\beta = 97.8$ wt% Sn and $C_{\text{eutectic}} = 61.9$ wt% Sn. Applying the appropriate lever rule expression for $W_\beta$

$$W_\beta = \frac{C_0 - C_{\text{eutectic}}}{C_\beta - C_{\text{eutectic}}} = \frac{C_0 - 61.9}{97.8 - 61.9} = 0.57$$

and solving for $C_0$ yields $C_0 = 82.4$ wt% Sn.
9.37 For a 30 wt% Zn-70 wt% Cu alloy, make schematic sketches of the microstructure that would be observed for conditions of very slow cooling at the following temperatures: 1100°C (2010°F), 950°C (1740°F), 900°C (1650°F), and 700°C (1290°F). Label all phases and indicate their approximate compositions.

Solution

The illustration below is the Cu-Zn phase diagram (Figure 9.19). A vertical line at a composition of 30 wt% Zn-70 wt% Cu has been drawn, and, in addition, horizontal arrows at the four temperatures called for in the problem statement (i.e., 1100°C, 950°C, 900°C, and 700°C).

On the basis of the locations of the four temperature-composition points, schematic sketches of the four respective microstructures along with phase compositions are represented as follows:
1100°C

$L$
(30 wt% Zn)

950°C

$L$
(32 wt% Zn)

$\alpha$
(27 wt% Zn)

900°C

$\alpha$
(30 wt% Zn)

700°C

$\alpha$
(30 wt% Zn)

$\alpha$

$\alpha$
The room-temperature tensile strengths of pure lead and pure tin are 16.8 MPa and 14.5 MPa, respectively.

(a) Make a schematic graph of the room-temperature tensile strength versus composition for all compositions between pure lead and pure tin. (Hint: you may want to consult Sections 9.10 and 9.11, as well as Equation 9.24 in Problem 9.64.)

(b) On this same graph schematically plot tensile strength versus composition at 150°C.

(c) Explain the shapes of these two curves, as well as any differences between them.

Solution

The (a) and (b) portions of the problem ask that we make schematic plots on the same graph for the tensile strength versus composition for lead-tin alloys at both room temperature and 150°C; such a graph is shown below.

(c) Upon consultation of the Pb-Sn phase diagram (Figure 9.8) we note that, at room temperature (20°C), about 1.5 wt% of Sn is soluble in Pb (within the α-phase region at the left extremity of the phase diagram). Similarly, only about 1 wt% of Pb is soluble in Sn (within the β-phase region at the left extremity). Thus, there will a solid-solution strengthening effect on both ends of the phase diagram—strength increases slightly with additions of Sn to Pb [in the α phase region (left-hand side)] and with additions of Pb to Sn [in the β phase region (right-hand side)]; these effects are noted in the above figure. This figure also shows that the tensile strength of pure lead is greater than pure tin, which is in agreement with tensile strength values provided in the problem statement.

In addition, at room temperature, for compositions between about 1.5 wt% Sn and 99 wt% Sn, both α and β phase will coexist, (Figure 9.8). Furthermore, for compositions within this range, tensile
strength will depend (approximately) on the tensile strengths of each of the \( \alpha \) and \( \beta \) phases as well as their phase fractions in a manner described by Equation 9.24 for the elastic modulus (Problem 9.64). That is, for this problem

\[
(TS)_{\text{alloy}} = (TS)_\alpha V_\alpha + (TS)_\beta V_\beta
\]

in which \( TS \) and \( V \) denote tensile strength and volume fraction, respectively, and the subscripts represent the alloy/phases. Also, mass fractions of the \( \alpha \) and \( \beta \) phases change linearly with changing composition (according to the lever rule). Furthermore, although there is some disparity between the densities of Pb and Sn (11.35 versus 7.27 g/cm\(^3\)), weight and volume fractions of the \( \alpha \) and \( \beta \) phases will also be similar (see Equation 9.6).

At 150°C, the curve will be shifted to significantly lower tensile strengths inasmuch as tensile strength diminishes with increasing temperature (Section 6.6, Figure 6.14). In addition, according to Figure 9.8, solubility limits for both \( \alpha \) and \( \beta \) phases increase—for the \( \alpha \) phase from 1.5 to 10 wt% Sn, and for the \( \beta \) phase from 1 to about 2 wt% Pb. Thus, the compositional ranges over which solid-solution strengthening occurs increase somewhat from the room-temperature ranges; these effects are also noted on the 150°C curve above. Furthermore, at 150°C, it would be expected that the tensile strength of lead will be greater than that of tin; and for compositions over which both \( \alpha \) and \( \beta \) phases coexist, strength will decrease approximately linearly with increasing Sn content.